AP Calculus AB Summer Assignment

In order to help us have the smoothest transition in to calculus we would like you to firm up some of your previous math skills, graphing calculator ability, and introduce you to some of the course material (don't worry you did see it in pre-calc).

Due: First Day of School

- There will be a quiz over this material the first Friday of school.
- Students who complete this assignment and do well on the quiz may earn up to 10 points of extra credit in the test/quiz category.
- This is your first homework grade, worth 30 points. Incomplete assignments will earn 0 points.
- You are encouraged to work with other students, but each individual student must have their own work.
- It is suggested that you work on this assignment in parts over the summer. It is NOT suggested to wait until August to complete this assignment.
- You may complete all your work on this paper. **Show ALL work! Box all answers.**
- A graphing calculator is required for some questions.
- Round answers to the nearest thousandth (this is the expected norm on the AP test and in class)
- Evaluate trigonometric functions in RADIANS unless otherwise indicated.
- Check and correct all questions in a different color then what you completed the problem in.
- Self-grade your assignment, and be honest in your efforts, you will receive full credit for attempting every problem (even if you get them all wrong).

Some Online Resources:

www.purplemath.com

www.kahnacademy.com

http://tutorial.math.lamar.edu/Classes/CalcI/CalcI.aspx

http://www.mastermathmentor.com/calc/RUReady.ashx

https://www.wolframalpha.com/

If you have any questions, you can contact us over the summer. Please email both of us and we will respond when we have the chance – we are in and out of town throughout the summer.

Have a great summer and see you in the fall!

Karen Gipson (gipsong@pcsb.org) and *Karen Smalley* (smalleyka@pcsb.org)

A. Solve Linear Equations

- Can you solve an equation requiring distribution, combining like terms, and clearing the fraction?
- Can you solve for a variable in a literal equation?

1. Solve for x.

$$\frac{1}{4} + \frac{2}{3}(4x + 9) = 0$$

2. Solve for h.

$$R = \frac{1}{2}t^2 h$$

B. Equation of a Line

- Can you write the equation of a line in point-slope, standard, and slope-intercept form?
- Can you determine the slope of a line given two points, a function, or a graph?
- Can you determine the intercepts of a line?

3. For the points (3, -4) and (-5, 2), write the equation of a line passing through the two points in point-slope.

4. Write the equation of a line parallel to $2x - 3y = 4$ and passing through $(8, 1)$ in point-slope form.

5. Write the equation of a line perpendicular to $y = -\frac{1}{3}x + 2$ and passing through (-2, 3) in point-slope form.

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C. System of Equations and Inequalities

- Can you identify the solution of a system of equations or a system of inequalities from a graph?
- Can you solve a system of equations by graphing, substitution, and elimination?
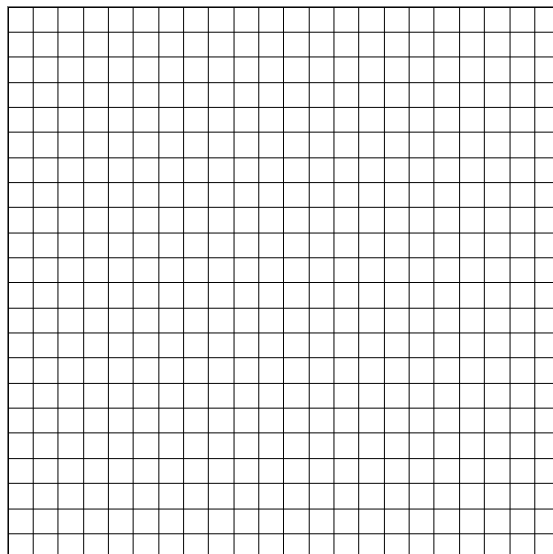
6. Solve for x and y. $\begin{cases} 5x + y = 48 \\ 4x + 5y = 9 \end{cases}$

7. Solve for x and y. $\begin{cases} 4y - 3x = 19 \\ 2y = 12x + 6 \end{cases}$
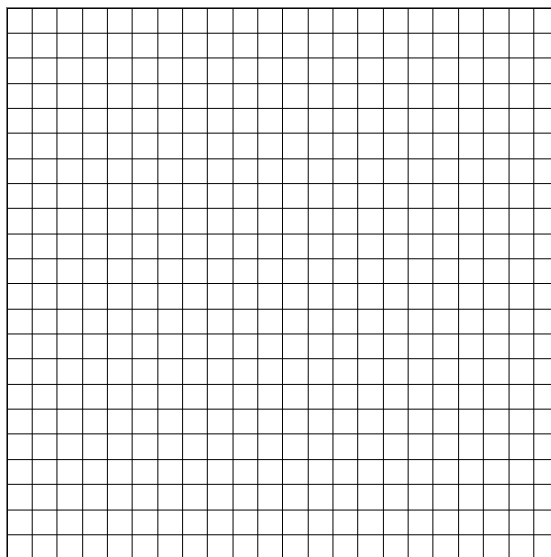
D. Piecewise Functions

- Can you graph piecewise functions?
- Can you evaluate piecewise functions?
- Can you determine the domain and range of piecewise functions?

8. Graph $y = \begin{cases} 4 \ if \ x < -4 \\ |x| \ if \ -4 \leq x \leq 4 \\ 4 \ if \ x > 4 \end{cases}$

9. Graph $y = \begin{cases} -5x + 2 \; if \; x < 2 \\ 2x - 2 \; if \; x \geq 2 \end{cases}$

10. Evaluate $h(x) = \begin{cases} x^2 - 3x + 7, \; x < -4 \\ 2x - 7, \; x = 7 \\ 8, \; x \neq 7 \; and \; x \geq -4 \end{cases}$

   a) $h(7)$ c) $h(-4)$

   b) $h(0)$ d) $h(-5)$

E. Domain and Range

- Can you determine the domain and range given the function or graph?

11. $f(x) = 7$

   Domain:

   Range:

12. $f(x) = \frac{5x-3}{2x+1}$

   Domain:

   Range:

13. $f(x) = \frac{8x}{x(x^2-49)}$

   Domain:

   Range:

14. $f(x) = 2^x - 3$

   Domain:

   Range:

15. $f(x) = |x - 4|$

   Domain:

   Range:

16. $f(x) = \sin(2x)$

   Domain:

   Range:

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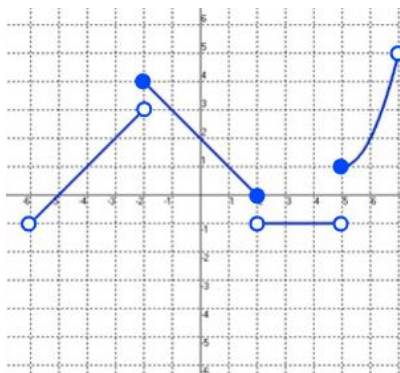17. $f(x) = \ln (x + 1)$

   Domain:

   Range:

18. $f(x) = \frac{2}{\sqrt{x+4}}$
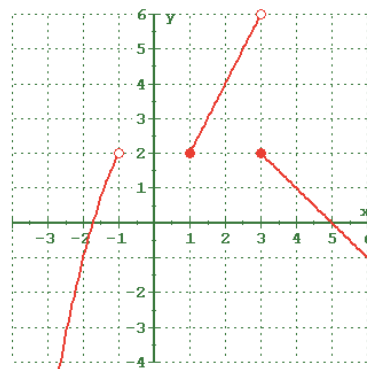
   Domain:

   Range:

19. Find the domain and range for the following piecewise function.

Domain:

Range:

20. Find the domain and range of for the following piecewise function.
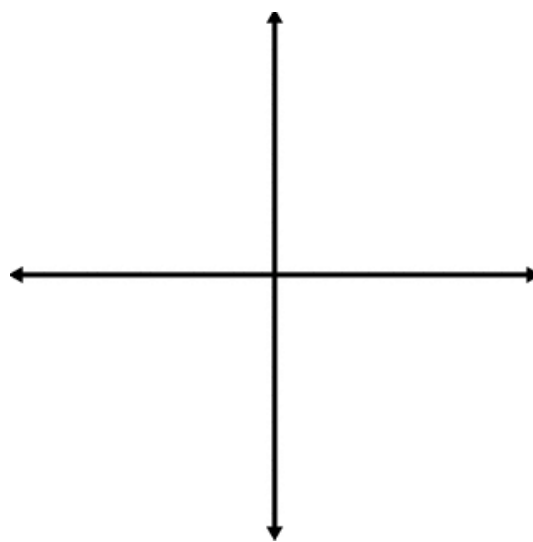

Domain:

Range:

F. Function Behavior

- Can you identify when a function is increasing and decreasing?

- Can you find the zeroes of a function?

- Can you identify points of discontinuity and name the types of discontinuity?
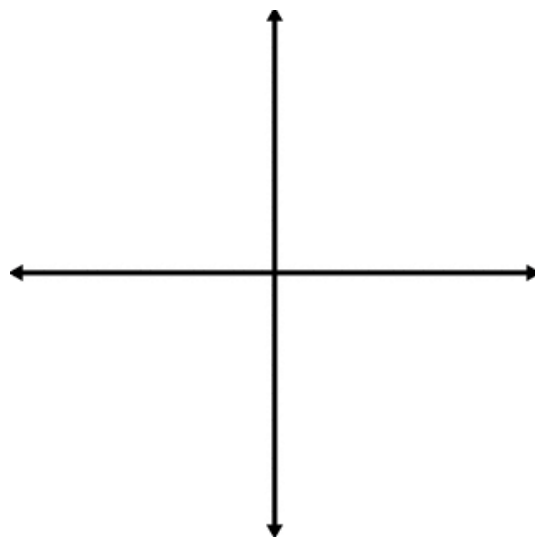
21. Sketch a function with the following characteristics

- A vertical asymptote at $x = 3$

- A jump at $x = -1$

- Passes through $(0, 2)$

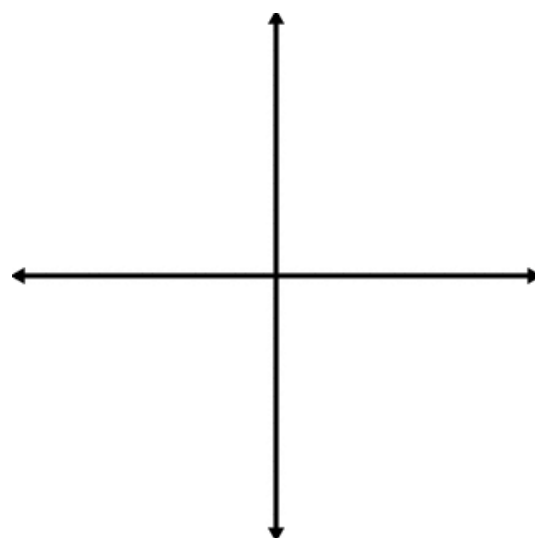- Decreasing on the interval $(-\infty, -1)$

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22. Sketch a function with the following characteristics

- A horizontal asymptote at $y = -4$
- A removable discontinuity at $x = 2$

23. Sketch a function with the following characteristics

- Domain is [-5, 6]
- Range is [-8, 2]
- The function is increasing on (-4, 0) only
- $f(0) = 2$

G. Factoring

- Can you factor a quadratic?

- Can you factor a polynomial completely?

24. Factor completely: $4x^2 - 28x - 32$

27. Factor completely: $4x^3 - 8x^2 - 25x + 50$

25. Factor completely $6x^2y - 24xy^3 + 8xy$

28. Factor completely: $64x^2 - 49$

26. Factor completely $3x^2 + 10x + 7$

H. Quadratics

- Can you solve a quadratic equation be factoring, square roots (if possible), completing the square, and the quadratic formula?

- Can you graph quadratic functions?

- Can you determine the domain and range of quadratic functions?

29. Solve by factoring.

$5x^2 + 14x + 14 = 2 - 2x$

For #41-43: Solve using square roots (if possible), completing the square, or quadratic formula.

30. Solve $121x^2 = 49$ 31. Solve $x^2 + 24x = -8$

I. Rational Expressions and Equations

- Can you simplify a rational expression?
- Can you add/subtract/multiply/divide rational expressions?
- Can you graph rational functions?
- Can you solve rational equations?

32. Simplify the following completely. State any restrictions on the domain.

$$\frac{x^2-4}{x^2+5x+6}$$

33. Simplify the following completely.

$$\frac{1}{2} + \frac{1}{2+h}$$

34. Simplify the following completely. State any restrictions on the domain.

$$\frac{\frac{1}{x-6}}{\frac{1}{x^2-36}}$$

35. Simplify the following completely.

$$\frac{4}{x} + \frac{x}{2} + \frac{3}{x^2}$$

36. Simplify the following completely. State any restrictions on the domain.

$$\frac{\frac{1}{a} - \frac{1}{b}}{\frac{2}{ab}}$$

37. Simplify the following completely. State any restrictions on the domain.

$$\frac{5 - \frac{1}{2x}}{\frac{3}{x-2}}$$

J. Exponents

- Can you simplify exponents?

- Can you write exponents in the form $x^{\frac{n}{m}}$ in radical form?

For #51-52: Simplify the exponent expressions so there are no negative or rational exponents.

38. $(3x^2 y^{-4})^3$

39. $(3x)(2y^2)(4x^{-3})(5y^0)$

For #53-54: Re-write the exponent expressions in radical form.

40. $4x^{\frac{1}{2}}$

41. $(5y)^{-\frac{3}{2}}$

For #55-56: Re-write the expression with rational exponents.

42. $\dfrac{2}{\sqrt{x+3}}$

43. $\sqrt[3]{\dfrac{1}{x}}$

K. Graphing

- Can you identify the parent function of quadratic, cubic, square root, $\dfrac{1}{x}$, sin, cos, tan, csc, cot, sec, e^x, ln(x), etc.?
- Can you identify translations of parent functions?

44. Match the following graphs (next page) with the function. DO NOT USE A CALCULATOR or DESMOS. Functions may be used once or not at all.

A) $y = e^x$	B) $y = \tan x$	C) $y = \sin x$	D) $y = -\ln x$	E) $y = \dfrac{1}{x-4}$	F) $y = \ln(x-3)$
G) $y = |x+3|$	H) $y = \cos x$	I) $y = e^x - 2$	J) $y = \dfrac{1}{3}x^3 - 6$	K) $y = \sec x$	L) $y = e^{-x}$
M) $y = -\dfrac{1}{x}$	N) $y = \cos(x+2)$	O) $y = (x-3)^2$	P) $y = x^3 - 6$	Q) $y = 2\sin x$	R) $y = \sqrt{x} + 4$
S) $y = 2(x-3)^2$	T) $y = \csc x$	U) $y = -|x+3|$	V) $y = \sqrt{-x} + 4$	W) $y = \dfrac{1}{x}$	X) $y = \ln x$
Y) $y = \cot x$	Z) $y = \dfrac{1}{x} + 3$				
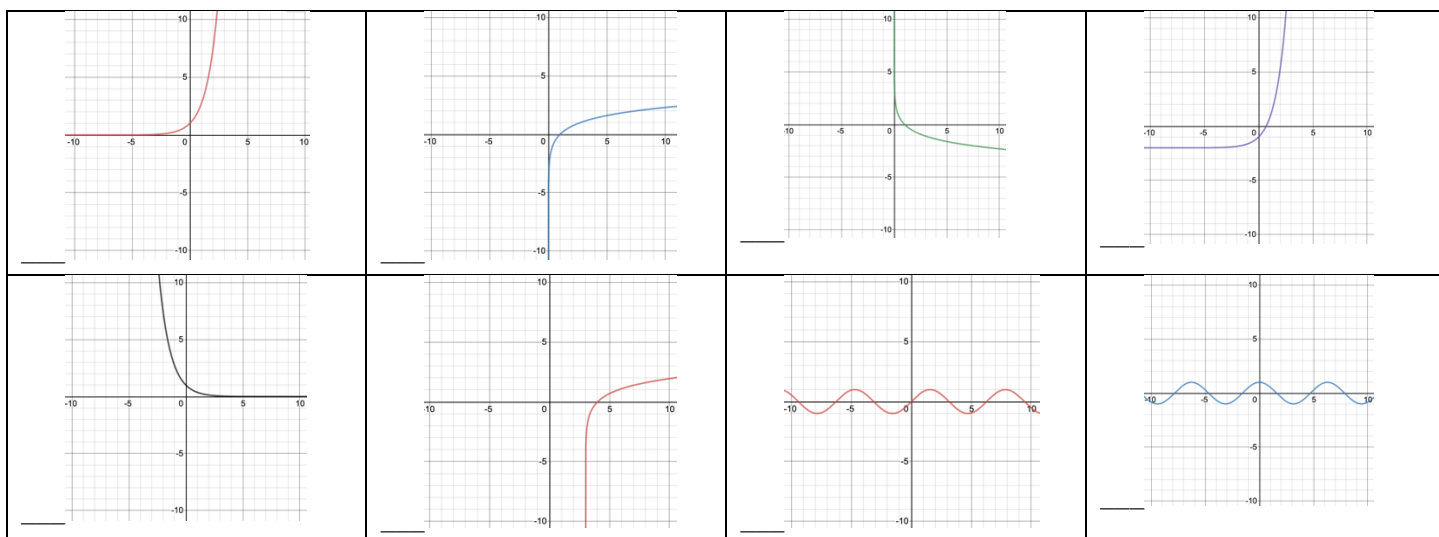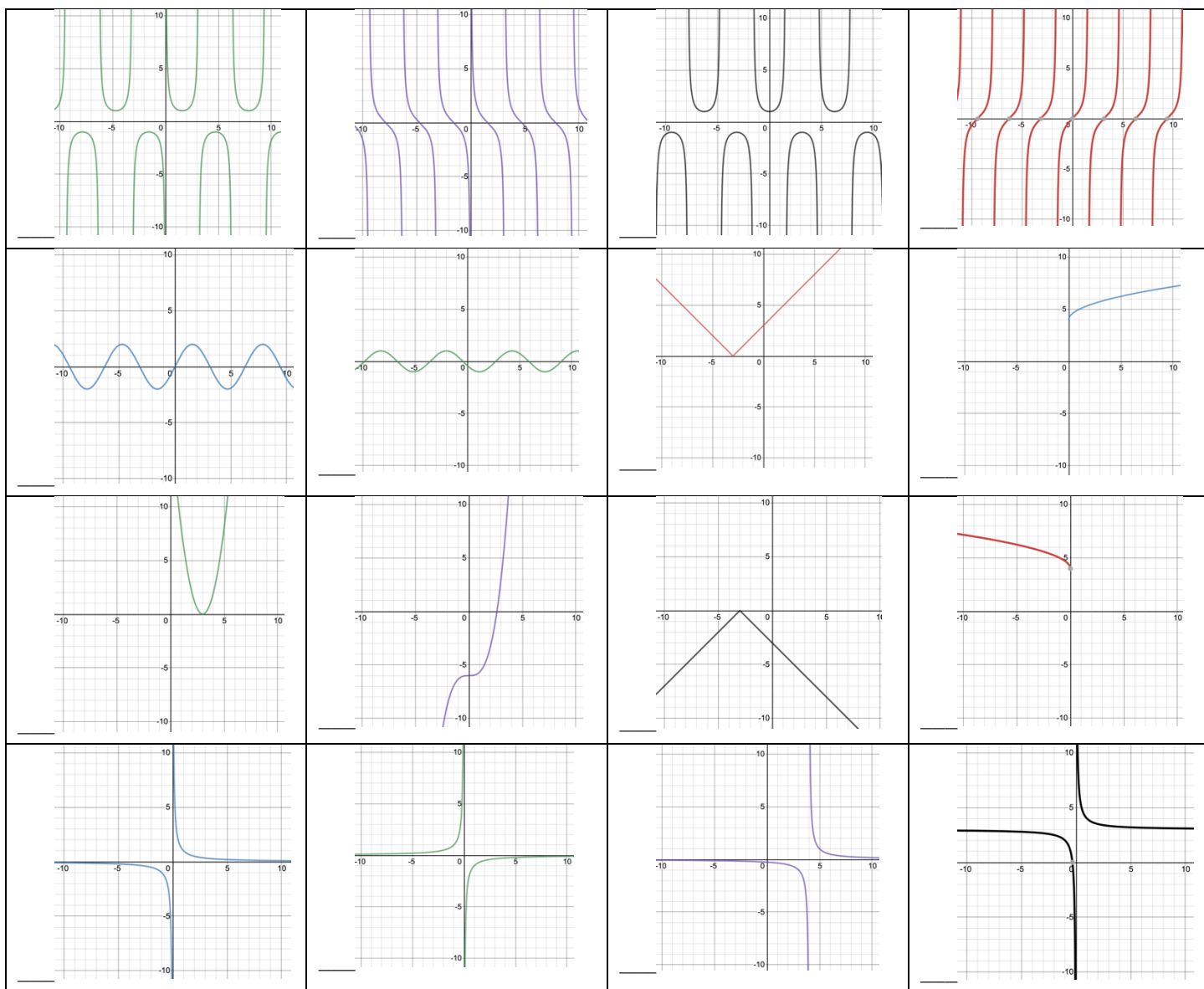
L. Exponential and Logarithmic Functions

- Can you graph exponential functions?
- Can you identify the domain and range of exponential and logarithmic functions?
- Can you simplify logarithms?
- Can you solve logarithmic and exponential equations (we will mainly work in base e)?

45. Use properties of logarithms to simplify:

a) $2\ln(x-3) + \ln(x+2) - 6\ln x$

b) $3\log x + 4\log y - 2\log z$

46. Use properties of logarithms to expand:

    a) $\ln(x^2 y^7)$
                                                                b) $\log(x-4)^2$

47. Express y in terms of x.

    $\ln y = x + 2$

48. Solve for x.

    $e^{2x} = 5$

50. Solve for x.

    $\ln(4 - x) = 3$
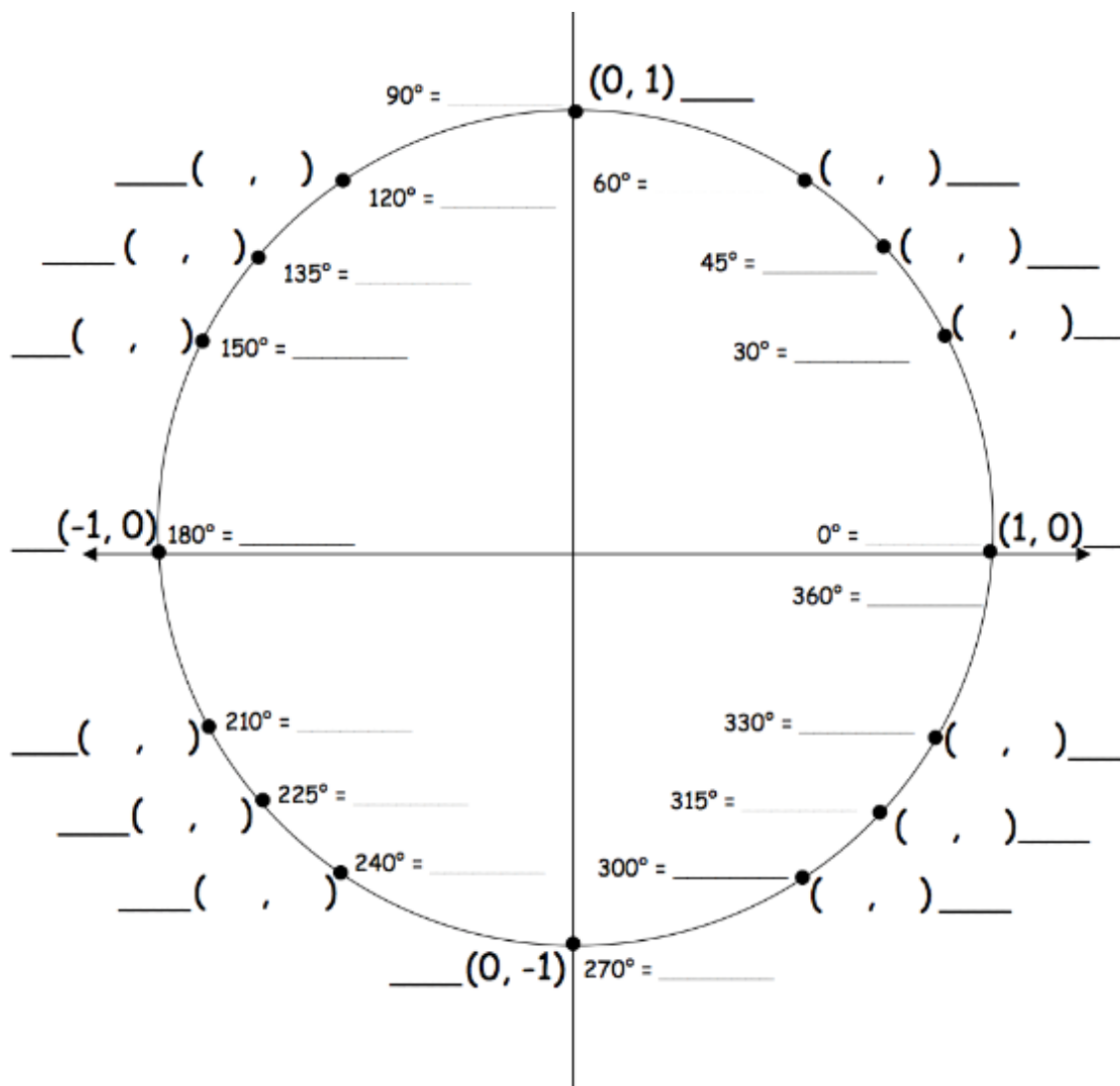
49. Solve for x.

    $4e^x + 5 = 8$

51. Solve for x.

    $\ln 6 + \ln x = 3$

M. Trigonometry

- Do you know the values of sin, cos, and tan for the unit circle?
- Can you graph the sin, cos, and tan graph and translations?
- Can you graph the $\sin^{-1} x, \cos^{-1} x, and \tan^{-1} x$ graphs and their translations?
- Can you complete proofs involving trigonometric identities?

52. Complete the following chart for the trigonometric values. Convert each of the degree measures to radians. Leave answers as radicals in simplest form (no decimals!). **You MUST know these values for Calculus.**

Degrees	Radians	sin	cos	tan
0	0			
30	$\dfrac{\pi}{6}$			
45				
60				
90	$\dfrac{\pi}{2}$	1	0	undefined
120				
135				
150				
180				
210				
225				
240				
270				
300	$\dfrac{5\pi}{3}$	$-\dfrac{\sqrt{3}}{2}$		
315				
330				
360				

90° = _____   (0, 1) _____

_____(,)   120° = _____   60° =   (,) _____

_____(,)   135° = _____   45° = _____   (,) _____

_____(,)   150° = _____   30° = _____   (,) _____

_____(-1, 0) 180° = _____   0° = _____   (1, 0) _____

360° = _____

_____(,)   210° = _____   330° = _____   (,) _____

_____(,)   225° = _____   315° = _____   (,) _____

_____(,)   240° = _____   300° = _____   (,) _____

_____(0, -1) 270° = _____

53. Evaluate $f(x) = \sin^2 x - \cos^2 x$ at $x = \frac{\pi}{2}$ (No calculator!)

54. Evaluate $g(x) = \frac{3 + \sin x}{\cos x}$ at $x = \frac{\pi}{3}$ (No calculator! No decimal answers! Simplify your answer)

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N. Geometry

- Do you know the area formulas for the following: circle, square, parallelogram, trapezoid?

- Do you know the volume formulas for the following: cone, cylinder, prism, pyramid?

There are no problems for this section. Please review the listed formulas.

O. Function Operations and Inverses

- Can you add/subtract/multiple/divide functions?

- Can you compose functions?

- Can you simplify $\frac{f(x+h)-f(x)}{h}$?

- Can you find the inverse of a function?

- Can you evaluate function notation?

55. Simplify $\frac{f(x+h)-f(x)}{h}$, where $f(x) = 2x + 3$.

56. Simplify $\frac{f(x+h)-f(x)}{h}$, where $f(x) = x^2$

For #76-80: Perform the following operations given $a(x) = 3x^2 - 1, b(x) = 2x + 7, c(x) = \frac{1}{x}$. State the domain and range.

57. $(a + b)(x)$

60. $b\big(c(x)\big)$

58. $a(x)b(x)$

61. $\frac{b(x)}{c(x)}$

59. $c\big(b(x)\big)$

62. Consider functions f and g shown on the graph below. Compute the following.
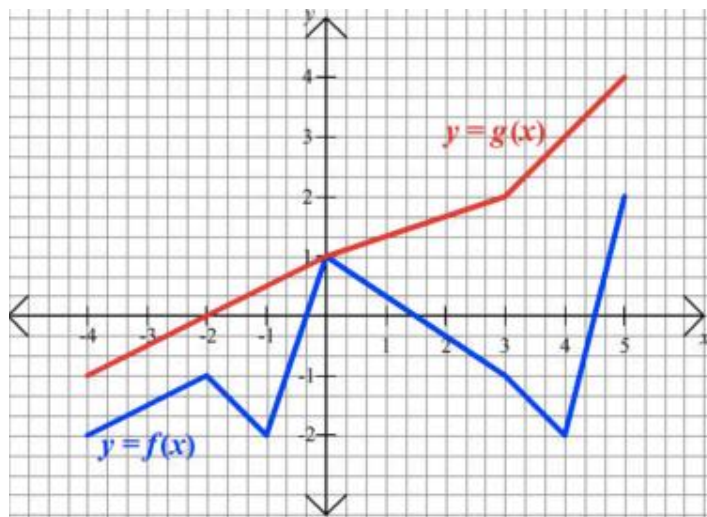
a) $f(4) + g(0)$

b) $f(g(-2))$

c) $f(3) - g(-4)$

d) $g(0) + 3$

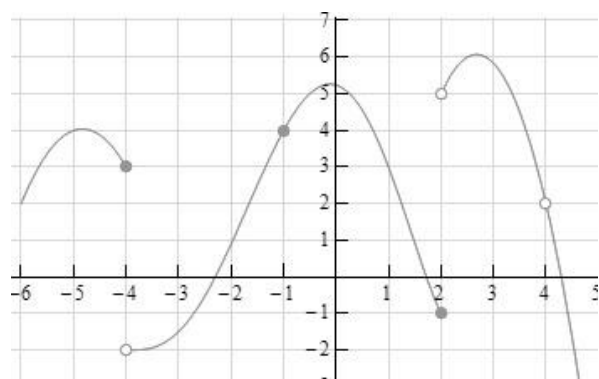e) Sketch the graph of $g(x) + 3$ on the graph above in GREEN.

f) Sketch the graph of $f(x + 2)$ on the graph above in ORANGE.

P. Limits

- Can you evaluate limits using graphs?
- Can you evaluate one sided limits?
- Can you evaluate limits algebraically?
- Can you determine if a function is continuous?

63. Given the graph of $f(x)$ evaluate the following limits and values

a) $\displaystyle\lim_{x \to -4^-} f(x)$

g) $\displaystyle\lim_{x \to 4^+} f(x)$

b) $\displaystyle\lim_{x \to -4^+} f(x)$

h) $f(-1)$

c) $\displaystyle\lim_{x \to -4} f(x)$

i) $f(2)$

j) $f(4)$

d) $\displaystyle\lim_{x \to -1} f(x)$

k) Is the function continuous at $x = 2$?

e) $\displaystyle\lim_{x \to 1} f(x)$

l) Is the function continuous at $x = -1$?

f) $\displaystyle\lim_{x \to 4^-} f(x)$

m) Is the function continuous at $x = 4$

64. Given the graph of $f(x)$ evaluate the following limits and values
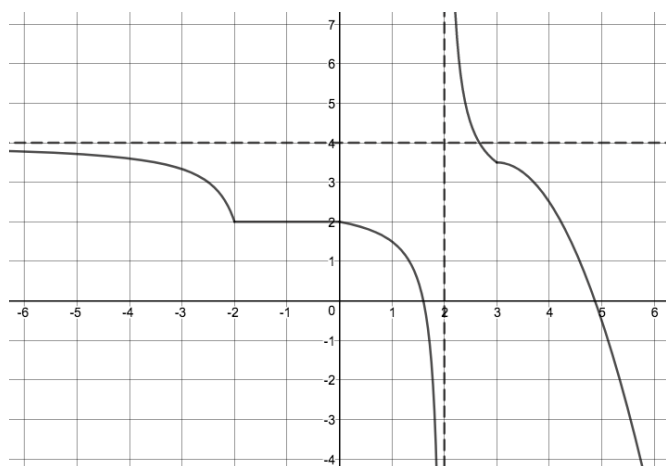
a) $\lim\limits_{x \to -2^-} f(x)$

b) $\lim\limits_{x \to -2^+} f(x)$

c) $\lim\limits_{x \to -2} f(x)$

d) $\lim\limits_{x \to -\infty} f(x)$

e) $\lim\limits_{x \to 2^-} f(x)$

f) $\lim\limits_{x \to 2^+} f(x)$

g) $\lim\limits_{x \to 0} f(x)$

h) $f(-2)$

i) $f(0)$

j) Is the function continuous at $x = 2$?

k) Is the function continuous at $x = -1$?

65. Evaluate the following limits analytically. Do NOT use a calculator or a graph.

a) $\lim\limits_{x \to -2} x^2 - 6$

b) $\lim\limits_{x \to \frac{\pi}{4}} \cos^2(2x)$

c) $\lim\limits_{x \to 2} \frac{x^2-4}{x-2}$

d) $\lim\limits_{x \to \infty} 3x$

e) $\lim\limits_{x \to \pi} \tan x$

f) $\lim\limits_{x \to \infty} \frac{2}{x}$

g) $\lim\limits_{x \to 3} \frac{x^2+2x-15}{x-3}$

Q. Calculator

- Can you use SOLVER and/or INTERSECT on your graphing calculator to find the zeroes of a function?
- Can you use your graphing calculator to find the zeroes of a function?
- Can you use your graphing calculator to find the intersection of two function?
- Can you use your graphing calculator to find the minimum/maximum of a function?
- Can you use the table function of your graphing calculator?
- Can you set an appropriate viewing window for a graph?

66. For the function $f(x) = x^3 - 3x^2 - x + 2$ (Use the graphing features of your calculator to complete these questions).

a) Sketch the graph

b) Determine the relative minimum(s) using the minimum function of the graphing calculator. (Round to the nearest thousandth, if necessary)

c) Determine the relative maximum(s) using the maximum function of the graphing calculator. (Round to the nearest thousandth, if necessary)

d) Determine the interval(s) when the function is increasing.

e) Determine the interval(s) when the function is decreasing.

f) Find the *x*-intercepts (zeroes) of *f(x)* using the zeroes feature of the graphing calculator.

g) Graph $g(x) = -x + 4$ and $f(x) = x^3 - 3x^2 - x + 2$. How many times do the functions intersect? Find the points of intersections using the intersection function of the graphing calculator.
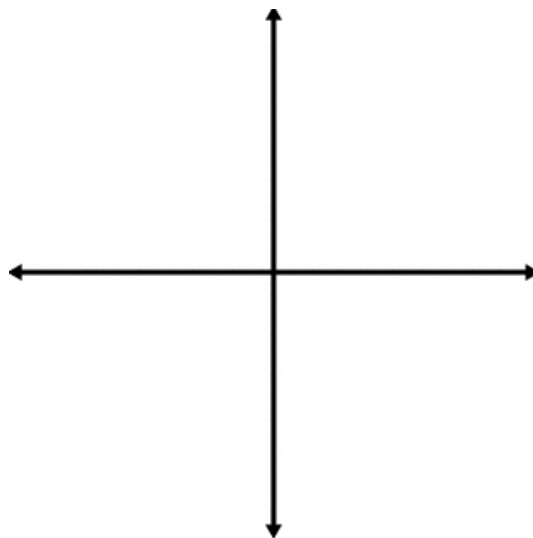
67. Find all real roots to the nearest 0.001 using the SOLVER or INTERSECTION feature of a graphing calculator.

a) $f(x) = 3\sin(2x) - 4x + 1 \; from \; [-2\pi, 2\pi]$ (in radian mode)

b) $f(x) = |x - 3| + |x| - 6$

68. Use your calculator to find the solution(s) to the following. Answers must be correct to 4 decimal places.

a) $e^{2x} + 4x = x^2 + 3$

b) $\sin(x + 3) = 3x^2 - 3$

R. Derivatives [This section is NOT required]

- Can you find the derivative of functions using the power rule, quotient rule, and product rule?
- Can you find the derivate of trigonometric functions?

69. Find the derivative of the following functions.

a) $f(x) = (3x^2 + 7)(x^2 - 2x + 3)$

e) $m(x) = \frac{(x^2 + x - 1)}{(x^2 - 1)}$

b) $g(x) = \frac{2}{(3x)^2}$

f) $c(x) = x^3 - 3x^2$

c) $h(\theta) = 2\theta - 3\sin\theta$

g) $h(x) = 6\sqrt{x} + 3\sqrt[3]{x}$

d) $k(x) = \sqrt{x}\sin x$